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Approximation Designs for Cooperative Relay Deployment in Wireless Networks

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***Abstract*—In this paper, we aim to maximize users' satisfaction by deploying limited number of relays in a target region to form a wireless relay network, and define the *Deployment of Cooperative Relay (DoCR) problem*, which is proved to be NP-complete. We first propose an $O(\delta \log n)$ approximation algorithm that utilizes the algorithms for budget weighted Steiner tree problem with novel position weighting assignment. We further propose a heuristic method to solve the DoCR problem releasing potential location constraint. Our extensive experiments indicate that the algorithms we propose can significantly improve the total satisfaction of the network. Furthermore, we establish a testbed using USRP to showcase our designs in real scenarios. To the best of our knowledge, we are the first to propose approximation algorithm for relay placement problem to maximize user satisfaction, which has both theoretical and practical significance in the related area.**

I. Introduction

With the increasing popularity of WLAN networks, most users in the public areas require high quality wireless service. Thus, providing better service for them becomes a challenging problem. However, owing to many factors like long transmission distance, limited Access Point (AP) coverage, fast signal attenuation, etc., the signal strength at some positions is not strong enough for users to access the Internet [1]. Wireless Relay is a signal forwarding device to amplify and forward the wireless signals received to users. Since it is tiny, convenient, and mobile, with excellent signal amplification effect, placing relays in multi-hop networks becomes an effective method to improve the system capacity, wireless coverage, and service quality [2]–[4].

In this paper, we investigate relay deployment problem in a target region with great number of users expecting wireless signals. Due to the limited transmission power, many users cannot receive wireless signal or are not satisfied about the signal strength. To make things better, we hope to place limited number of relays and maximize the users' satisfaction of the network service. We refer this problem as the *Deployment of Cooperative Relay (DoCR) problem*.

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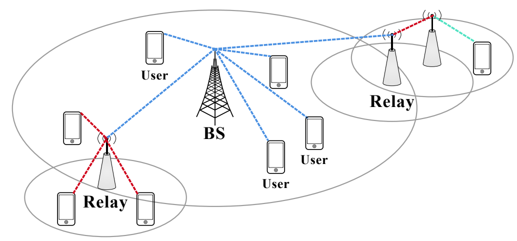

Fig. 1. Illustration of a wireless relay network

In wireless networks, the *target coverage* problem has been paid more attentions [5], [6], which is similar to DoCR problem. However, they did not take network connectivity into consideration. With the concept of relay emerging, many works turned to keep connectivity and extend the coverage through deployment of relays [7], [8], some paid attention to the energy consumption [9]–[11], where rare of research discussed users' satisfaction. Cui et al. [12] first proposed "quality of cooperation", but this function ignored the distance between users and relays. Additionally, their proposed heuristic algorithms lack theoretical bound analysis.

Correspondingly, in this paper, we define user satisfaction function comprehensively. Owing that relays cannot generate wireless signal itself, each relay must have a path to the base station. Fig. 1 illustrates such requirements. Next, we define the DoCR problem formally and propose an approximation *Relay Effective Deployment Algorithm (REDA)* and a new heuristic *Gradient-Descent Based Algorithm (GDBA)* releasing the potential location constraint. REDA utilizes the algorithms for budget weighted Steiner tree problem with novel position weighting assignment. Through the algorithm analysis, its approximation ratio reaches $O(\delta \log n)$, where δ is the maximum node degree and n is the number of potential relay locations.

Finally, we perform extensive experiments and the numerous results indicate that our algorithms can significantly improve the total satisfaction of the network. Furthermore, we establish a testbed using Universal Software Radio Peripheral (USRP) to showcase our designs in real scenarios. In all, to the best of our knowledge, we are the first to provide approximation algorithms for relay placement problem to maximize user satisfaction, which has both theoretical and practical significance in the related area.

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The rest of the paper is organized as follow. Problem definition is given in Sec. II. Sec. III introduces REDA, with approximation ratio analysis. Sec. IV describes GDBA. Extensive experiments and testbed implement are demonstrated in Sec. V and Sec. VI. Finally, Sec. VII concludes this paper.

II. Problem Statement

In this section, we give the network model and formulate the DoCR problem, then prove its NP-completeness, for which we can hardly develop polynomial time optimal algorithms.

A. Network model

We can model a wireless relay network as a directed graph. Firstly, assume there are $U = \{u_1, u_2, \cdots, u_m\}$ users in a target region Ω, which is continuous, flat, and has no obstacles. Then, finding the best positions for relays become a continuous optimization problem, which is hard to estimate the optimal solutions and evaluate the accuracy of solutions.

To simplify the problem, we give two assumptions: (1) users' locations are known (via some positioning systems); and (2) we predefine n potential relay locations, denoted as $R = \{r_1, r_2, \cdots, r_n\}$, one of which should be the base station. In this way we convert our problem into a combinatorial optimization problem. The denser the R is, the more accurate our model could reflect the original problem.

We denote relays and the base station as *signal sources*, since they can provide wireless service for users. They should form a connected graph, which will be defined in Subsec. II-B. We denote $D \subseteq R$ as the selected location set of relays. $|D|$ is the cardinality of D, or the number of relays we can deploy.

B. Problem Definition

We will consider the problem in which all the users and relays are homogeneous. Let function $s(r_j, u_i)$ denote the satisfaction degree for user u_i after receiving the signal from relay r_j. $s(\cdot)$ relates to many factors such as the distance between users and relays, the transmission process, and etc.

Definition 1 (Service Radius)**.** *Service radius, denoted as d_s, is a distance threshold for a signal source r_i. Only users whose distance to r_i is less than d_s could receive its wireless signal.*

Definition 2 (Connected Pair)**.** *Given a potential location subset $A \subseteq R$, the connected pair function $p_A(u_i)$ returns a signal source in A, from which user u_i receives the strongest signal. From the view of satisfaction, $p_A(u_i) = \arg\max_{r_j \in A} s(r_j, u_i)$. We let $p(u_i) = p_R(u_i)$ by default.*

Definition 3 (Cumulative Satisfaction)**.** *The cumulative satisfaction for any subset $A \subseteq R$, denoted as $s(A)$, is defined as the sum of user satisfaction if we place relays at A. Say, $s(A) = \sum_{i=1}^{m} s(p_A(u_i), u_i)$.*

Definition 4 (Communication Radius)**.** *Two signal sources can forward the wireless signal to each other within a communication radius d_c. Usually, we assume $d_c = 2d_s$.*

Since the wireless signal cannot be generated by itself, the relay has to receive signal from other signal sources. Thus, the locations where we deploy relays must form a connected subgraph, meaning that each r_i has a path to the base station.

Definition 5 (Relay Connectivity)**.** *Given potential position set R, we generate a graph $G = (R, E)$, where the distance between r_i and r_j, $(r_i, r_j) \in E$. Relay connectivity means that the induced graph $G[D]$ for the selected relays is connected.*

Now, we are able to define the *Deployment of Cooperative Relay (DoCR) problem* as follow.

Definition 6 (DoCR Problem)**.** *Given a user set U, a potential location set R, a satisfaction function $s(\cdot)$, a relay number constraint k, and a service radius d_s, the DoCR problem is to find a subset $D \subseteq R$ such that $|D| \leq k$, the subgraph induced by D is connected, and $s(D)$ is maximized.*

Owing to the limited number of relays, the wireless serviced cannot provided in the whole region. Thus, we may not satisfy all the users in the region. Meanwhile, because the devices of users are different, their satisfaction degree functions vary according to their devices' abilities of receiving signal. Our goal is to provide most users better wireless service.

Theorem 1. *DoCR problem is NP-complete.*

Proof. We can reduce the DoCR problem from a known NP-complete problem, named the *budgeted set cover* (BSC) problem [13]. Given a set of elements E and a collection of subsets $\mathbf{S}$, the goal of BSC is to select at most L sets such that the number of elements covered by these sets is maximized.

Suppose we have an algorithm A_D to compute DoCR problem. For any instance $\{\mathbf{S}, E, L\}$ of the BSC problem, we can construct a complete graph where each node represents a subset in $\mathbf{S}$. Every element has a unit satisfaction $s(e_i) = 1$ when it is covered. The limit number k is the budget L. Then we get an instance for the DoCR problem, which can be computed by A_D. The output can be seen as the solution to BSC problem as well, since we do not have the connectivity problem for a complete graph. Therefore, according to Cook's reduction, the DoCR problem is NP-complete. □

III. Relay Effective Deployment Algorithm

In this section, we propose *Relay Effective Deployment Algorithm (REDA)* to solve DoCR, which has two stages.

Basically, we will scan each potential relay position r_i in turn to judge whether to select it. Note that a user's satisfaction could be changed once we add a new relay into the selected set. It means different selection orders might influence the results greatly. Thus, we hope to assign a fixed weight to each relay, regardless of their selection orders. Correspondingly, in the first stage, we define a weight function and design a greedy algorithm to assign fixed weight for each r_i.

When each r_i has a fixed weight, DoCR will turn into the *Budget Node-Weighted Steiner Tree (BNWST)* problem [14]. Thus in the second stage, we can implement any approximations for BNWST to solve DoCR.

Together, the detail of REDA is presented in Alg. 1.

Algorithm 1: Relay Effective Deployment Alg. (REDA)

Input: An instance of DoCR, $\langle U, R, d_s, k, s(\cdot)\rangle$
Output: The selected relay location set $D \subseteq R$.
1 $\mathcal{A} \leftarrow \emptyset$;
2 **while** $|\mathcal{A}| \leq n$ **do**
3   Select $r_i \in R \backslash \mathcal{A}$ which maximizes $s_{\mathcal{A}}(r_i)$;
4     $w(r_i) \leftarrow s_{\mathcal{A}}(r_i)$; $\mathcal{A} \leftarrow \mathcal{A} \cup \{r_i\}$;
5 Apply an α-approximation on instance $\langle \mathcal{A}, w\rangle$ to obtain a budget node weighted Steiner tree T ;
6 **return** $D = V(T)$

A. Stage 1: Weight Assignment

Firstly, let us give some definitions for weighting.

Definition 7 (Residual Satisfaction)**.** *Consider a selected relay location set $B \subseteq R$, a relay r_j and a user u_i. The residual satisfaction of u_i to r_j based on B is defined as $s_B(r_j, u_i) = \max\{s(r_j, u_i) - s(p_B(u_i), u_i), 0\}$. Then the residual satisfaction for a relay is $s_B(r_j) = \sum_{u \in F(r_j)} s_B(r_j, u)$. $\forall A \subseteq R$, $s_B(A) = \sum_{u \in F(A)} \max\{s(p_A(u), u) - s(p_B(u), u), 0\}$.*

Definition 8 (Relay Selection Sequence)**.** *Let $\mathcal{A} = \{r_a \to r_b \to \cdots \to r_{|\mathcal{A}|}\}$. It denotes a selected relay sequence from potential relay set R. $\mathcal{A} \subseteq R$ means that $\cup\{r \in \mathcal{A}\} \subseteq R$.*

We design a greedy algorithm to assign weights for each r_i. In each iteration, we select a r_i with maximum $s_{\mathcal{A}_{i-1}}(r_i)$, where $\mathcal{A}_{i-1}$ is the ordered sequence in the previous iteration. Then we define a weight function w for locations as $w(r_i) = s_{\mathcal{A}_{i-1}}(r_i)$. The algorithm terminates when each r_i has a weight assigned. The detail is shown in Line 1-4 of Alg. 1.

After running Line 1-4 of Alg. 1, $\mathcal{A} = \{r_1, r_2, \cdots, r_n\}$, with the order we selected. Let $\mathcal{A}_i = \{r_1, r_2, \cdots, r_i\}$ be a subsequence with the first i elements. Now, let us analyze the weight assignment. Let $OPT \subseteq R$ be the optimal selection of DoCR problem, and $opt = s(OPT)$. Two concepts will be used in the analysis.

First, we claim that we can find a subsequence $\mathcal{B} \subseteq \mathcal{A}$ such $|\mathcal{B}| \leq k$ and $w(\mathcal{B}) > (1 - e^{-1})opt$. We can select $\mathcal{B}$ by Alg. 2, which scans each location in $\mathcal{A}$ sequentially. If $r_i \in \mathcal{A}$ connects to OPT, we add it into $\mathcal{B}$ until $|\mathcal{B}| = k$, where OPT is the optimal selection of DoCR problem. This selection scheme is not same with traditional selection scheme in the submodular theory. Actually, OPT is unknown, so we could never implement Alg. 2. We just use this idea to justify the effect of our algorithm.

When Alg. 2 terminates, assume $\mathcal{B} = \{r_{j_1}, r_{j_2}, \cdots, r_{j_k}\}$.

Lemma 1. $s_{\mathcal{A}_{j_{i-1}}(r_{j_i})} \geq \frac{s_{\mathcal{A}_{j_{i-1}}}(OPT \backslash \mathcal{A}_{j_{i-1}})}{|(OPT \backslash \mathcal{A}_{j_{i-1}})|}$, $1 \leq i \leq k$.

Proof. According to the greedy policy in Alg. 1, the residual satisfaction of r_{j_i} is the maximum among all other unselected locations in OPT. □

Lemma 2. $s(OPT \cup \mathcal{A}_{j_i}) - s(\mathcal{A}_{j_i}) \geq s(OPT) - w(\mathcal{B}_i)$, $\forall i = 1, \cdots, k$.

Algorithm 2: Selection of $\mathcal{B}$

Input: $\mathcal{A}$ got from Alg. 1 with new weights w, k
Output: A subsequence $\mathcal{B} \subseteq \mathcal{A}$
$\mathcal{B} \leftarrow \emptyset$; $i \leftarrow 1$;
while $|\mathcal{B}| < k$ **do**
    if *r_i is connected to OPT* **then** $\mathcal{B} \leftarrow \mathcal{B} \cup \{r_i\}$;
    $i \leftarrow i + 1$;
return $\mathcal{B}$

Proof. To prove this lemma, we consider each user's satisfaction function from the user perspective rather than the relay perspective. We suppose $A = OPT \cup \mathcal{A}_{j_i}$. We have

$$\begin{aligned} & s(A) - s(\mathcal{A}_{j_i}) \\ = & \bigcup_{u \in F(A)} s(p_A(u), u) - \bigcup_{u \in F(\mathcal{A}_{j_i})} s(p_{\mathcal{A}_{j_i}}(u), u) \\ = & \bigcup_{u \in F(OPT)} s(p_A(u), u) - \bigcup_{u \in F(OPT \cap \mathcal{B}_i)} s(p_{\mathcal{A}_{j_i}}(u), u) \\ \geq & \ s(OPT) - w(\mathcal{B}_i) \end{aligned}$$

Therefore, we have the inequality below:

$$w(\mathcal{B}_i) - w(\mathcal{B}_{i-1}) = s_{\mathcal{A}_{j_{i-1}}}(r_{j_i}) \geq \frac{s_{\mathcal{A}_{j_{i-1}}}(OPT \backslash \mathcal{A}_{j_{i-1}})}{|OPT \backslash \mathcal{A}_{j_{i-1}}|}$$

$$\geq \frac{s(OPT \cup \mathcal{A}_{j_{i-1}}) - s(\mathcal{A}_{j_{i-1}})}{k} \geq \frac{opt - w(\mathcal{B}_{i-1})}{k}$$

The first equality follows the definition of the weight function. The first inequality is proved by Lemma 1. The third inequality comes from Lemma 2.

Let $a_i = opt - w(\mathcal{B}_i)$. Then we can get

$$\begin{aligned} & w(\mathcal{B}_i) - w(\mathcal{B}_{i-1}) \geq \frac{opt - w(\mathcal{B}_{i-1})}{k} \\ \Leftrightarrow \quad & a_{i-1} - a_i \geq \frac{a_{i-1}}{k} \Leftrightarrow a_i \leq (1 - \frac{1}{k})a_{i-1} \\ \Rightarrow \quad & a_k \leq (1 - \frac{1}{k})^k a_0 \leq e^{-1} a_0 \Leftrightarrow w(\mathcal{B}_k) \geq (1 - e^{-1})opt \end{aligned}$$

Now, we get that the final output of Alg. 2 satisfies $s(\mathcal{B}_k) \geq w(\mathcal{B}_k) \geq (1 - e^{-1})opt$, where $\mathcal{B} \subseteq \mathcal{A}$ and $|\mathcal{B}| \leq k$. In addition, the selected set $\{OPT \cup \mathcal{B}\}$ is connected with $|OPT \cup \mathcal{B}| \leq 2k$.

B. Stage 2: Node Selection

In this stage, the output of relay selection sequence $\mathcal{A}$ and its new weight is regarded as the input of the BNWST problem, whose definition is shown in Def. 9.

Definition 9 (Budget Node-weighted Steiner Tree problem (BNWST))**.** *Given an undirected graph $G = (V, E)$, a cost function $c : V \to \mathbb{R}^+$, a profit function $\pi : V \to \mathbb{R}^+$, and a budget L, the BNWST problem is to find a subtree T of G such that $c(T) = \sum_{v \in V(T)} c(v) \leq L$ and $\pi(T) = \sum_{v \in V(T)} \pi(v)$ are maximized.*

From the definition, we can get that the weight function is the profit function for BNWST. If the distance between two locations is less than d_c, then there is an edge and the cost function is a unit function. k is another input to represent the budget. Thus, the output of BNWST problem is a feasible solution to the DoCR problem. If there exists an α-approximation for BNWST, let us analyze the approximation ratio of DoCR.

Lemma 3. *Any tree T with $2n$ nodes can be decomposed into at most δ subtrees such that each subtree has at most n nodes, where δ denotes the maximum degree of tree T.*

Proof. We denote all the nodes in T as $V(T)$. For any node $u \in V(T)$, we can regard T as a tree rooted at u, denoted by T^u. Let $N(u)$ be the neighbor set of u. For each nodes $v \in N(u)$, denote the subtree of T^u containing all descendants of v including itself as T^u_n. It is a branch of T^u.

Among all nodes of $V(T)$, we choose a node u such that $\max\{|T^u_v| : v \in N(u)\}$ is as small as possible. If there is a subtree T^u_v such that $|T^u_v| \geq n$, then we have $|T - T^u_v| \leq n$. We can regard subtree $T - T^u_v$ as a branch of T^v. Thus, it leads to $\max\{|T^v_x| : x \in N(v)\} \leq \max\{|T^u_x| : x \in N(u)\}$. It is a contradiction with the choice of u. Hence, $\max\{|T^u_v| : v \in N(u)\} \leq n$. Owing that it is impossible for every subtree T^u to have n nodes, the root node u can be contributed to the subtree T^u_v who has least nodes among these subtrees.

Therefore, there are at most δ subtrees of T^u such that every subtree has at most n nodes. □

Theorem 2. *The output D of Alg. 1 is a connected relay locations with satisfaction $s(D) \geq \frac{1}{\delta\alpha}(1-\frac{1}{e})opt$ and $|D| \leq k$.*

Proof. Suppose T^* is an optimal Budgeted Node-Weighted Steiner Tree and T is the Budgeted Node Weighted Steiner tree computed in Alg. 1. Then $D = V(T)$ is a feasible solution to the DoCR problem.

First, according to Def. 7, we can get $s(D) \geq w(T)$. Then, we have proved that the selected set $\{OPT \cup \mathcal{B}\}$ is connected and it is a spanning tree T'. According to Lemma 3, we can decompose the spanning tree into at most δ subtrees such that each subtree has at most k nodes. Thus, there exists a subtree T'' of T' such that

$$|T''| \leq k \text{ and } w(T'') \geq \frac{1}{\delta} w(T') \geq \frac{1}{\delta} w(\mathcal{B})$$

Thus, combining the lemmas, we have

$$\begin{aligned} s(D) &\geq w(T) \geq \frac{1}{\alpha} w(T^*) \geq \frac{1}{\alpha} w(T'') \\ &\geq \frac{1}{\delta\alpha} w(T') \geq \frac{1}{\delta\alpha} w(\mathcal{B}) \geq \frac{1}{\delta\alpha}(1 - \frac{1}{e})opt \end{aligned}$$

The theorem has been proved. □

For the BNWST problem above, Bateni et al. [15] proposed an $O(\log |V|)$-approximation, where V is the set of nodes of graph. We can implement this algorithm in the second step of Alg. 1. Then we have Theorem 3.

Theorem 3. *Let $n = |R|$ be the number of potential relay locations, the DoCR problem has an $O(\delta \log n)$-approximation.*

IV. Gradient-Descent Based Algorithm

In this section, we release the constraint for discrete potential relay locations, which means that relays can be deployed anywhere in the region. We propose the *Gradient-Descent Based Algorithm (GDBA)* to solve DoCR.

In our problem, the main idea of GDBA is as follows. First, a base station is deployed at a random place. A covered region, where the relays can receive wireless signal from the base station, is generated. Then we deploy a relay in the covered region and try to find the direction for this relay where the total satisfaction of our network can increase most. Next the relay move a designated step along this direction until its step size is lower than the threshold of step size. After moving, a relay has already been deployed. We begin to deploy next relay until $k-1$ relays have been deployed.

The main challenge of GDBA is to find the best direction. To achieve this, owing that the residual satisfaction is not a continuous function, we need to define the partial derivation of the residual satisfaction function.

Definition 10. *(Partial Derivation of Residual Satisfaction Function): According to Def. 7 about the residual satisfaction function, we consider a selected location set A, a relay r_j and a user u_i. Suppose we provide a movement $\vec{x}$ for r_j, and define the partial derivation of the residual satisfaction function as*

$$\frac{\partial s_A(r_j, u_i)}{\partial \vec{x}} = \begin{cases} \frac{\partial s(r_j, u_i)}{\partial \vec{x}} & , s(r_j, u_i) > s(p_A(u_i), u_i) \\ 0 & , s(r_j, u_i) \leq s(p_A(u_i), u_i) \end{cases}$$

According to Def. 10, the relay should move to the direction which increases the satisfaction of the networks most, i.e. $\triangle\vec{x} = \max \frac{\partial s_A(r_j)}{\partial \vec{x}}$. We denote the step size as δ, determining how long the relay moves at each time.

The step size δ is initialized as δ_0. When the location is out of the covered region or the received satisfaction is lower than the original location's, we adjust the step size as $\delta = \frac{1}{2}\delta$ and try to move again. The relay node R_j stops moving until it is less than the threshold $\bar{\delta}$. The smaller the threshold is, the more accurate the result is. When all these K relay nodes have been deployed, the total satisfaction is the output of our algorithm, presented in Alg. 3.

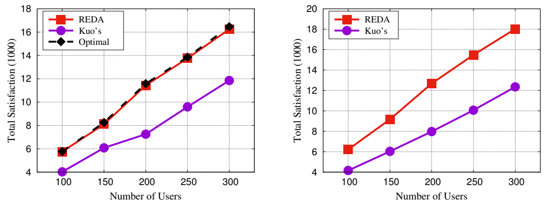
V. Numerical Experiments

In this section, we perform simulations using C++ and MATLAB to study the performance of the proposed algorithms.

We design two simulations, A and B, with 20 and 100 potential relay locations respectively, shown in Tab. I, to present the performance of REDA through variation of user number and relay number. With the variation of potential location numbers, we compare the performance of REDA with GDBA without potential locations constraints. In addition, we can get the optimal solution to DoCR problem in Simulation A.

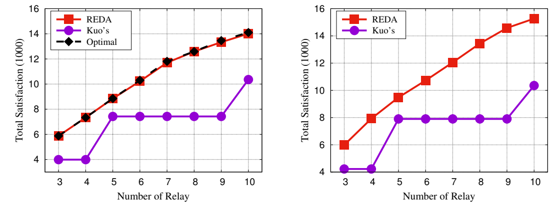
Assumption 1. *The satisfaction function of a user is concave.*

According to the law of diminishing marginal utility, the growth speed of satisfaction will be slow down with the



(a) 20 potential relay locations (b) 100 potential relay locations

Fig. 2. Impact of User Number



(a) 20 potential relay locations (b) 100 potential relay locations

Fig. 3. Impact of Relays Number

Algorithm 3: GDBA

Input: An instance of DoCR problem without potential locations, a threshold of step size $\overline{\delta}$ and the initial step size δ_0

Output: The total satisfaction of the network

$s(U) \leftarrow 0$; $\delta \leftarrow \delta_0$; $j \leftarrow 1$; $\mathcal{A} \leftarrow \emptyset$

Randomly deployment r_1 in the region

while $j \leq k$ **do**

  Randomly deploy r_j at the feasible location

  while $\delta \geq \overline{\delta}$ **do**

    $\triangle\vec{x} = \max \frac{\partial s_{\mathcal{A}}(r_j)}{\partial \vec{x}}$; $r_j = r_j + \delta \frac{\triangle\vec{x}}{|\triangle\vec{x}|}$

    if $s_{\mathcal{A}}(r_j) \leq 0$ || r_j *is out of covered region* **then**

      Move r_j back to original location; $\delta \leftarrow \frac{1}{2}\delta$

  $\mathcal{A} \leftarrow \mathcal{A} \cup r_j$, $j \leftarrow j+1$

return $s(\mathcal{A})$

TABLE I
SIMULATION PARAMETERS

Parameter	Region	m	n	k	d_s
Simulation A	$100m \times 100m$	$100-300$	20	$3-10$	20
Simulation B	$100m \times 100m$	$100-300$	100	$3-10$	20

enhancement of wireless signal received. Assumption 1 can be justified [12]. Therefore, we denote the satisfaction function of a user as

$$s(r_j, u_i) = 100 * (1 - (\frac{||r_j - u_i||}{20})^4),\ ||r_j - u_i|| \leq 20$$

where $||r_j - u_i||$ represents the Euclidean distance between r_j and u_i. It is a concave function within domains.

A. The Impact of User Number

We first study the effect of user number to the performance of REDA's results of the DoCR problem when the relay number is set to 7. Kuo's algorithm is proposed by [16], which mainly uses submodular greedy algorithm. We can see from the results, characterized in Fig. 2, that the total satisfaction is growing with increasing number of users in this region.

Fig. 2(a) demonstrates the performance of REDA with 20 potential locations, which is close to the optimal solution, and much better than Kuo's algorithm. Fig. 2(b) exhibits the performance of REDA when there are 100 potential relay locations. It is also much better than Kuo's algorithm. The advantages of our algorithm are demonstrated.

B. The Impact of Relays Number

The impact of relay number is also studied in the Simulation A and B. As shown in Fig. 3, there are 200 users randomly distributed in the region. With the increasing number of potential relay locations, REDA has more choices to deploy relays, leading to the growing of satisfaction. The performance is better than Kuo's algorithm, mainly owing to the deficiency of it. Fig. 3(a) depicts that the performance of REDA is close to the optimal solution.

C. The Impact of Potential Relay Location Number

We still consider the same region with 200 users distributed. The potential relay location number varies from 50 to 300 and we can select 10 locations as signal sources. The other parameters are not changed. We apply GDBA into this scene without the potential relay locations. Owing to its property of randomness, the solution is always varied. We execute the program 100 times and pick the maximum one as the output.

As shown in Fig. 4, lacking potential relay locations, the output of GDBA is a horizontal line. With the increasing number of potential relay locations, REDA has more choices. When the number of potential relay locations is more than 250, the result of REDA exceed GDBA's. Although GDBA can deploy relays anywhere, it is constrained by the initial locations and movement process. Randomness influences the performance of GDBA greatly. The location constrain would weaken gradually with the increasing number of potential relay locations and the advantage of an approximation algorithm is displayed gradually.

VI. TESTBED EXPERIMENT

In this section, we establish an experimental testbed using USRPs. In this experiment, we use USRPs to represent base stations, relays and users. The experiment is conducted in an outdoor wireless environment. Our model is established in a continuous and flat plate. We choose the campus in Shanghai Jiao Tong University as our experiment field, where there are many gathering activities of students. Therefore, it is an ideal place to implement our experiment.

In the field experiment, we use Ettus Research USRPs N210 with SBX daughterboard, VERT2450 antennas and employ GNU Radio v3.7.10 to program them. To simulate real Wi-Fi environment, the center transmit frequency of the base station is set as 2.437 GHz, which is the frequency of Wi-Fi channel 6. The transmit gain is 10 dB and bandwidth is $40MHz$ To

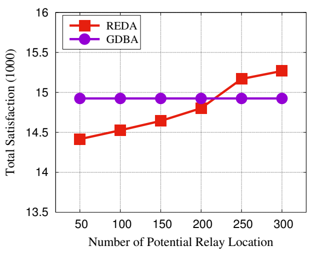


Fig. 4. Impact of Relay Location Number

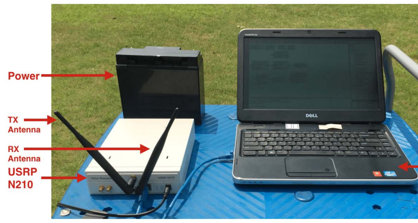


Fig. 5. Illustration of Relay

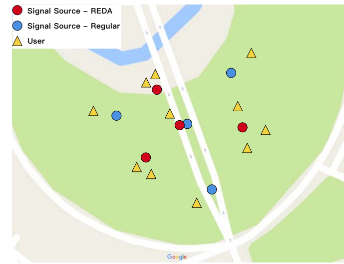
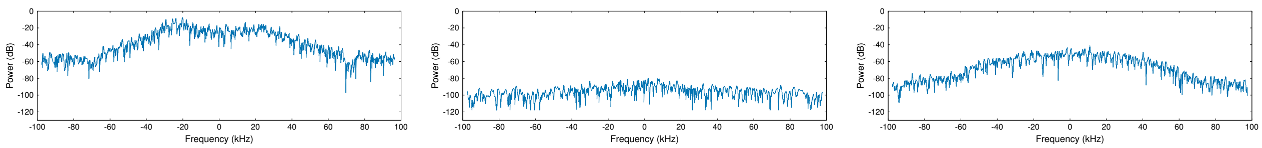


Fig. 6. Experimental Scene



(a) Transmiter

(b) Before using Relay

(c) After using Relay

Fig. 7. Waveforms Comparison between before and after Using Relay

avoid interference, the center transmitting frequency of each relay is 5 MHz greater than the receiving frequency.

In this experiment, we design a file transmission system and utilize the percentage ratio between receive rate and transmission rate to define user satisfaction function. We fit the satisfaction function by testing several time in the campus.

The design illustration of relays is shown in Fig. 5, which consists of a laptop running Ubuntu 16.04, an USRP N210 with two antennas and a portable power. To verify the effectiveness of wireless relays, we focus on the waveform of the file transmission system in Fig. 7. The strength difference of the waveform before and after using relay demonstrates that the effectiveness of wireless relays is manifest.

There are randomly 12 users distributed in the campus lawn, as shown in Fig. 6. First, we consider the regular deployment strategy, denoted by blue nodes. The middle one is the base station. Then we apply distribution of users as input to REDA and get the solution, denoted by red nodes. According to the satisfaction function mentioned above, we calculate the total satisfactions of two deployment strategies. The former one is 362.8 and the latter one reaches 751.6. Although the coverage of REDA deployment may not be larger than regular strategy's, it can determine the location based on users' distribution. Thus, it can provide better service for users in the region, showing the effectiveness of REDA in reality.

VII. Conclusion

In this work, we have studied the *Deployment of Cooperative Relay (DoCR)* problem in the wireless relay network, aiming at maximizing the total satisfaction of users. One $O(\delta \log n)$ approximation algorithm and one heuristic algorithm are proposed to solve DoCR problem. Extensive simulations report two algorithms can significantly improve the networks performance. In addition, we establish a testbed using USRPs. We design a reality field experiment in the campus and get the superiority of our algorithms. It is the first to investigate relay placement problem with both approximation analysis and practical significance in the related work.

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